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Limits on the accuracy of force sensing at short separations due to patch potentials

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Recent theoretical developments in gravitational physics have motivated experimental searches for violations of Newton's inverse square law of gravity at small separations. There has also been considerable theoretical and experimental progress in establishing the Casimir effect. These two classes of experiments and others in fundamental physics measure the forces between polycrystalline metals at micron-order separations, and are thus susceptible to forces due to electrostatic patch fields on their surfaces. We develop the theory for the patch force power spectra which provides the necessary tools to estimate the magnitude of random patch signals. We apply our results to two experiments intending to measure nonNewtonian gravitational signals using the isoelectronic technique where the mean effect of patches is nullified by design but random patch signals may still spoil the measurement sensitivity. Our results help gauge the sensitivity limitations of real experimental setups and the analytical formulas we derive suggest useful strategies for engineering and minimizing undesired patch effects in future experiments.

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The presence of patch potentials on metallic surfaces has important implications in various experiments, including measurements of gravity on elementary particles [1], heating in ion traps [2,3], the physics of Rydberg atoms [4,5], precision tests of general relativity in space [6,7], measurements of the Casimir force [8–12], and searches for hypothetical forces [13–25]. Patches lead to interactions that limit the accuracy and precision of such experiments. Within the context of force sensing between metallic samples, the theoretical modeling has focussed on quantifying the mean and sample-to-sample fluctuations of the patch effect [26–31], but an in-depth analysis of the patch force/torque power spectrum for time-independent surface potentials is lacking. Here, we develop such a theory, and apply it to the analysis of two ongoing experiments, one being performed at Birmingham [32], and another one at Indiana University-Purdue University Indianapolis (IUPUI) [33], that search for violations of Newton's inverse square law and new hypothetical forces using isoelectronic methods.

Isoelectronic setups provide an ideal platform to clarify the concept of random systematic patch effects because the mean patch effect is canceled out by construction, but spatial variations of the surface potential may still detrimentally affect the ability to constrain new physics. The isoelectronic measurement scheme utilizes a flat source mass with inhomogeneous mass density which is covered with a sufficiently thick metal coating to electrically screen the underlying materials. This has two important consequences: Both (i) the Casimir interaction and (ii) the mean patch force between the coated sample and a force probe are independent of the location of the latter at fixed separation distance. However, force signals that depend on lateral displacements of the probe can arise not only from the sought-after mass-dependent interactions between the probe and the underlying inhomogeneous mass density, but from spatially dependent patch force fluctuations as well. Hence, although the isoelectronic technique eliminates Casimir and mean patch forces, it is still possible that patch effects contribute random signals to the measurements. Such patch effects are systematic in nature, meaning that their influence cannot be removed by averaging. In order to quantify the magnitude of patch systematics, we compute the electrostatic torque and force power spectrum.

We start with the solution for the potential between two infinite planes located at $z = 0$ and $z = D$ with electrostatic potentials $V_1(\mathbf{y})$ and $V_2(\mathbf{y})$, respectively (here, $\mathbf{y}$ are coordinates defined in the xy-plane attached to each plate) [34]. In this geometry the potential takes the form $V(\mathbf{x}, z) = \int d^2y \int \frac{d^2k}{4\pi^2} \frac{e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}}{\sinh kD} [V_1(\mathbf{y}) \sinh k(D-z) + V_2(\tilde{\mathbf{y}}) \sinh kz]$, and the corresponding electrostatic interaction energy is [35]

$$\mathcal{E} = \frac{\varepsilon_0}{2} \int \frac{d^2k}{(2\pi)^2} \int d^2y \int d^2y' \frac{k e^{i\mathbf{k}\cdot(\mathbf{y}-\mathbf{y}')}}{\sinh kD} \times [(V_1(\mathbf{y})V_1(\mathbf{y}') + V_2(\tilde{\mathbf{y}})V_2(\tilde{\mathbf{y}}'))e^{-kD} - (V_1(\mathbf{y})V_2(\tilde{\mathbf{y}}') + V_1(\mathbf{y}')V_2(\tilde{\mathbf{y}}))]. \quad (1)$$

Here, $k = |\mathbf{k}|$ and D is the separation between the plates. Note that the argument of V_2 is given by the vector

$\tilde{\mathbf{y}} = \mathbf{R}_\phi \cdot (\mathbf{y} - \mathbf{a}) + \mathbf{b}$, which has been introduced for later convenience. $\mathbf{R}_\phi$ is an orthogonal rotation matrix, and $\mathbf{a}$ and $\mathbf{b}$ are vectors representing translations which parametrize the changes in the potential $V(\mathbf{x}, z)$ for all possible ways the two plates can be rotated and translated with respect to one another (see Fig. 1). The generalized force between the plates is defined as $\mathcal{F}_\xi(\zeta) = \partial\mathcal{E}/\partial\xi$ [36], where ξ is the conjugate variable to the force, and ζ is the coordinate being modulated. Note that the rotation angle ϕ, the displacement vectors $\mathbf{a}$ and $\mathbf{b}$, and the plane-plane separation D generally depend upon ζ. The z-component of the torque as a function of rotations about $\mathbf{a}$ can be obtained from $\mathcal{F}_\phi(\phi) = \partial\mathcal{E}/\partial\phi$, where ϕ plays the role of ξ and ζ, and $\mathbf{b} = -\mathbf{a}$. As another example, the lateral force in the x-direction as a function of shifts in the y-direction can be obtained from $\mathcal{F}_x(\zeta) = \partial\mathcal{E}/\partial x$, with $\phi = 0$, $\mathbf{a} = \mathbf{0}$, and $\mathbf{b} = x\hat{x} + \zeta\hat{y}$. Additionally, Eq. (1) may be used to compute the force between a sphere and a plane when the radius of the sphere R is much greater than the sphere-plane separation D, the condition for validity of the proximity force approximation (PFA). In this case the force is given by $\mathcal{F}_{\rm sp}(\zeta) = 2\pi R\mathcal{E}/A$, where A is the plate area. The power spectrum of the generalized force is defined as

$$S_{\mathcal{F}_\xi}[\beta] = \frac{1}{\Delta}\int_\Delta d\sigma e^{i\beta\sigma}\mathcal{F}_\xi(\zeta+\sigma)\mathcal{F}_\xi(\zeta), \tag{2}$$

where Δ denotes the "length" over which the measurement is performed, such as a time interval or a rotation angle, and the coordinate β represents the "frequency" which is conjugate to ζ; e.g. β would be a spatial frequency when changes in ζ parametrize relative translations of the plates. In the case of periodic displacements of the samples Δ is given by the period of the motion, e.g. for relative rotations $\Delta = 2\pi$ [37].

As an example we compute the power spectrum for the lateral force in the x-direction and the normal sphere-plane force as a function of lateral shifts in the x-direction. In this case there is no relative rotation of the planes; thus $\tilde{\mathbf{y}}$ is $\mathbf{y} + x\hat{x}$ ($R_\phi = 1$, $\mathbf{a} = 0$, $\mathbf{b} = x\hat{x}$, $\zeta = x$). For the lateral force, $\partial/\partial\xi$ is given by $\partial/\partial x$, and we use the PFA for the sphere-plane force. The terms which are quadratic in the potentials of a single plate in Eq. (1) are independent of the displacement $x\hat{x}$ and therefore only the cross terms between $V_1(\mathbf{y})$ and $V_2(\mathbf{y})$ contribute to the force power spectrum at finite spatial frequency ($\beta = k_x$). By making a change of variables in $V_2(\tilde{\mathbf{y}})$: $\tilde{\mathbf{y}} \to \mathbf{y}$ and taking a x-derivative of the energy $\partial\mathcal{E}/\partial x$ (with $\phi = 0$, $\mathbf{a} = \mathbf{0}$, and $\mathbf{b} = x\hat{x}$), or by multiplying by the appropriate factor for the PFA, we find

$$\left.\begin{matrix}\mathcal{F}_x(x)\\ \mathcal{F}_{\rm sp}(x)\end{matrix}\right\} = \varepsilon_0\int\frac{d^2k}{(2\pi)^2}\left\{\begin{matrix}-ik_x\\ \frac{2\pi R}{A}\end{matrix}\right\}\frac{ke^{ik_x x}}{\sinh kD}V_1[-\mathbf{k}]V_2[\mathbf{k}], \tag{3}$$

where we have introduced the two-dimensional spatial Fourier transform of the surface potentials, $V_{\rm a}[\mathbf{k}] = \int d^2ye^{-i\mathbf{k}\cdot\mathbf{y}}V_{\rm a}(\mathbf{y})$. Provided these are known, the forces $\mathcal{F}_x(x)$ and $\mathcal{F}_{\rm sp}(x)$ and their power spectra can be computed via Eqs. (2) and (3).

Without a precise knowledge of the surface potentials, we can estimate the power spectra by making some statistical arguments based on the general characteristics of the layout of the patch potentials. We assume that the surface potentials on the plates are statistically uncorrelated, that the geometry of the patch layout is rotationally and translationally invariant (in a statistical sense), and that the typical patch size is much smaller than the area of interaction. These assumptions allow the product of surface voltages to be written in terms of a two-point voltage correlation function: $V_{\rm a}(\mathbf{x})V_{\rm b}(\mathbf{x}') \to C_{\rm ab}(r)$, where $r \equiv |\mathbf{x} - \mathbf{x}'|$. In the following we keep the derivations as general as possible by retaining dependence upon $C_{\rm ab}(r)$, which could be supplied by surface potential measurements. However, when additional insight can be gleaned from an explicit form for $C_{\rm ab}(r)$, we approximate $C_{\rm ab}(r) \approx \delta_{\rm ab}V_{\rm rms}^2\exp(-\lambda r/\ell)$, where $V_{\rm rms}^2$ is the variance of the patch potential voltages, ℓ is the typical patch "diameter," and $\lambda = 1.9\sqrt{4/\pi}$ (see [5] for details of Monte Carlo simulations of patch distributions, where such a form for the correlation function was used), and both plates have the same correlation properties. While capturing the salient features of many models previously used in the literature [5,29], this approximation allows for an analytical derivation of the force power spectrum [38].

The lateral force power spectrum is given by

$$\begin{aligned}S_{\mathcal{F}_x}[k_x] &= \frac{\varepsilon_0^2 A}{\pi\,\Delta}\int_0^\infty dk_y\frac{k_x^2k^2}{\sinh^2 kD}|C[\mathbf{k}]|^2\\ &= 4\pi\varepsilon_0^2V_{\rm rms}^4\ell^4\frac{A}{\Delta}\int_0^\infty dk_y\frac{k_x^2k^2}{\sinh^2 kD}\frac{\lambda^2}{(k^2\ell^2+\lambda^2)^3},\end{aligned} \tag{4}$$

where A is the plate's area, and where we have used $V_i[\mathbf{k}]V_j[-\mathbf{k}'] = 4\pi^2\delta^2(\mathbf{k} - \mathbf{k}')\delta_{ij}C[\mathbf{k}]$. In the remainder of the manuscript we adapt the example calculation of the power spectrum given in Eq. (4) to estimate the magnitude of systematic force signals which will be present in two experiments using the isoelectronic technique [32,33].

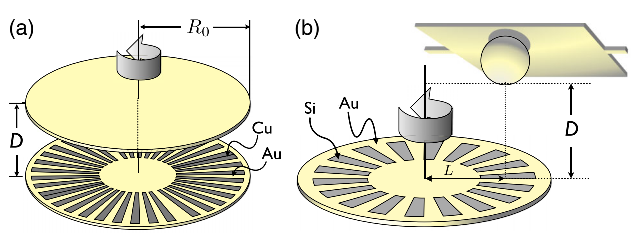

FIG. 1 (color online). (a) Setup of the torsion pendulum Birmingham experiment, and (b) the torsion balance IUPUI experiment. For clarity, the gold coating on top of the mass-density modulations of both disks is not shown. The underside of the top disk in (a) is also density modulated.

The Birmingham experiment and patch torque: This experiment searches for violations of the inverse square law of gravity by measuring the torque acting between two parallel disks [32]. As shown schematically in Fig. 1(a), two identical disks (radius $R_0 = 40$ mm, thickness 10 μm) are fabricated with a pattern of 2048 pairs of gold/copper spokes, and overlaid with a gold film (thickness 1 μm, not shown in Fig. 1(a)) for isoelectronic measurements. The 2048 mass pairs are grouped into 16 sectors of alternating phase to provide a further modulation of the signal. The lower disk, the test mass, is attached to a torsion balance with a superconducting suspension and the upper disk, the source mass, is attached to a micropositioner. The spacing between the gold surfaces is nominally $D = 13$ μm. The experiment has been designed to achieve a sensitivity of 10^{-17} N $\cdot$ m after an integration period of 1 day. The experimental procedure comprises the measurement of the torque as a function of the rotation angle, which should be angle periodic with a frequency determined by the mass-density modulation. Therefore, the magnitude of the patch effect relevant to the experiment can be calculated by evaluating the power spectrum of torque fluctuations at the frequency of the density modulation.

We take the patch torque as a function of the relative angle to be given approximately by $\mathcal{F}_\phi(\phi) \approx (R_0/2)\mathcal{F}_x(R_0\phi/2)$. The power spectrum peaks at values $k_x = 2m/R_0$ (m is an integer) [39], so that $S_{\mathcal{F}_\phi}[m] \approx (R_0/2)S_{\mathcal{F}_x}[k_x = 2m/R_0]$. The relevant value of m is 2048 because this corresponds to the angular frequency of the mass-modulated signal. We plot the magnitude of the expected torque fluctuations in Fig. 2(a) for $V_{\rm rms} = 4.5$ mV, which is close to the expected voltage variance for contaminated gold surfaces [40]. Theoretical calculations estimate $V_{\rm rms} = 45$ mV for clean gold surfaces [41].

The IUPUI experiment and patch force: In this experiment a Au-coated sapphire sphere (radius $R = 150$ μm) is glued to a microtorsional oscillator (MTO) [33]. The sphere-MTO assembly is brought into close proximity to a rotating disk, as schematically shown in Fig. 1(b). The apex of the sphere is at a distance $L = 1$ cm from the axis of rotation. The disk is the source mass formed by $2p$ ($p = 200$) sectors that subtend the same angle, which are alternately made of Si and Au. All the sectors are covered by a uniform layer of Au (thickness 200 nm), thick enough to make the setup isoelectronic. The disk is set to rotate at an angular frequency $\Omega = \omega_r/p$, where $\omega_r \approx 2\pi \times 300$ Hz is the resonant frequency of the MTO-sphere assembly [42]. The experiment is designed to achieve a sensitivity of 0.1 fN with an integration time of 1000 s. The experimental procedure comprises measuring the normal force on the MTO at its resonant frequency ω_r as a function of the disk-sphere separation D.

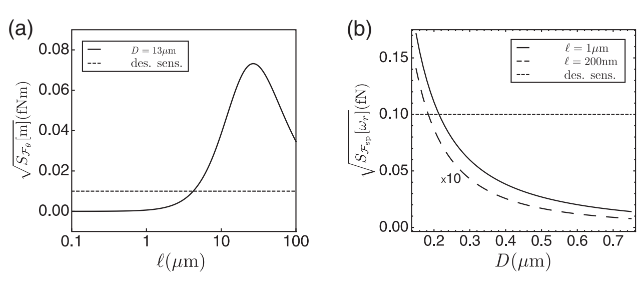

FIG. 2. (a) Patch torque power spectrum for the Birmingham experiment, and (b) patch force power spectrum for the IUPUI experiment. Both calculations are performed with $V_{\rm rms} = 4.5$ mV. Fig. 2(b) shows two curves corresponding to different patch sizes: one given by the thickness of the gold film, $\ell = 200$ nm (dashed line) and multiplied by 10 to be seen at the shown scale, and another for 1 μm size patches. The horizontal short dashed lines show the design sensitivity of the two setups.

The patch force adds a systematic signal to the measurement at the resonance frequency of the MTO. We estimate the magnitude of this effect by computing the force power spectrum using the PFA. The sphere will only detect a small portion of the plate centered at the point of closest approach with an area given approximately by $A_{\rm eff} \approx 2\pi RD$ [31]. The relative speed of the plate across the effective area of interaction changes by 0.3% and so we approximate the force as a function of relative displacement between the sample and probe using Eq. (3), where the displacement in x is replaced by the time-dependent position $x = L\Omega t$.

We estimate the normal patch force power spectrum from Eq. (2) by taking $\zeta \to t$ and approximating the surface potentials in terms of their correlation properties. This results in the sphere-plane force power spectrum,

$$S_{\mathcal{F}_{\rm sp}}[\omega_r] = \frac{2\epsilon_0^2 R}{\Omega L D}\frac{\Omega}{2\pi}\int_0^\infty dk_y \frac{k^2}{\sinh^2 kD}|C[\mathbf{k}]|^2|_{k_x=\frac{\omega_r}{\Omega L}}, \qquad (5)$$

evaluated at the MTO's resonance frequency. By taking the square root of $S_{\mathcal{F}_{\rm sp}}[\omega_r]$, we estimate the magnitude of the systematic patch force fluctuations at the frequency where the sought-after gravity signal should be present in Fig. 2(b). An important observation is that the patch force variance depends upon the sphere's distance from the disk's center of rotation, L. Therefore, the influence of patches can be ruled out if one measures the variance of the force signal, assuming no other systematic signals are present, for two different values of L, and their ratio is unity. Random patch signals restrict the level to which hypothetical forces can be constrained. These limits are determined when the magnitude of the random patch force matches the strength of the hypothetical gravitational signal that can be parametrized using a Yukawa correction to the gravitational potential, $V_{\rm grav} = -(Gm_1m_2/r)(1 + \alpha e^{-r/\lambda})$, between two point masses of mass m_1 and m_2, where G is the gravitational constant and r is the separation between the masses. We present our results in Fig. 3 alongside the data from several other groups, representing the current limits on Yukawa-like corrections to gravity, as a basis for

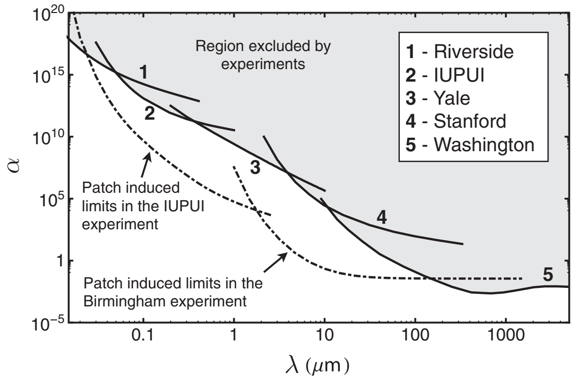

FIG. 3. Constraints on hypothetical new forces. The black dashed lines represent the limits with which hypothetical forces can be constrained due to the patch effects in the Birmingham and IUPUI experiments. We choose the typical patch size as the thickness of the gold film for the two experiments: $\ell = 1\ \mu$m for Birmingham, $\ell = 200$ nm for IUPUI. $V_{\rm rms}$ is set to 4.5 mV. As a basis of comparison we also indicate the current published constraints on Yukawa-like corrections to gravity: 1. [43–46], 2. [17,47,48], 3. [24], 4. [19], and 5. [20].

comparison. For the computation shown in Fig. 3 we take $V_{\rm rms} = 4.5$ mV; the typical patch size is set to the gold film thickness, 1 μm for the Birmingham apparatus and 200 nm for the IUPUI experiment. It is worth mentioning that for experiments performed in ultraclean environments at ultrahigh vacuum $V_{\rm rms}$ may take on its predicted value for clean surfaces (45 mV) which would increase the random patch force magnitude by a factor of 100.

The results presented in Fig. 2 and the relation $S_{\mathcal{F}_\xi} \sim V_{\rm rms}^4$ suggest two strategies to minimize the adverse effects imposed by patch potentials: one is to prepare the samples with judiciously chosen patch sizes, and the second is to reduce $V_{\rm rms}$. The first of these strategies could be implemented by preparing samples using thermal evaporation; in [49] it was observed that the size of the crystallites depends upon the ratio of the metal's melting point, T_m, to the temperature of the substrate, T_s. In the high temperature regime, $T_s > 0.5T_m$, the size of the crystallites is approximately equal to the film thickness. The second strategy may be realized as a consequence of surface contamination [40,50].

As another example, we have performed a similar analysis for the previous isoelectronic experiment done at IUPUI in 2005 [17]. In this experiment the inhomogeneous source mass was deposited on a MTO and the sphere was moved from regions of high to low mass density and back, following a periodic motion $z(t) = D + \delta D \cos \omega_z t$ normal to the MTO and $x(t) = x_0 \cos \omega_x t$ parallel to its axis. The modulation frequencies summed to the resonant frequency ω_r of the MTO. The experimental noise was observed to be 0.3 fN with an integration time of 1000 s. The experiment measured a nonvanishing signal (see Fig. 3 of [17]) that was not attributed to a new hypothetical force but to a differential Casimir force arising from a small variation in the height of the coated sample on the MTO. Even in the absence of such height variation, random patch forces could have contributed to the measured signal in Fig. 3 of [17]. We estimated the relevant patch force variance [51] and found that the signal observed in [17] may possibly be due to patches with sizes $< 18\ \mu$m with $V_{\rm rms} = 4.5$ mV.

In summary, we have computed the electrostatic patch force and torque power spectra, which provide the necessary tools to estimate the adverse effects of patches in a variety of setups and to understand and circumvent their problematic attributes. Our analysis shows that the random systematic signals due to patches decrease with smaller root-mean-square patch voltage fluctuations and can be lowered by appropriately tailoring the typical patch size. These two characteristics of the patch force power spectrum may be used as strategies to reduce the influence of patch potentials, for example via suitable chemical treatment of surfaces. Once the characteristics of surface potentials on polycrystalline metals is better understood, the techniques developed here will make precise calculations of patch effects possible.

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[37] In the case that the samples are periodically displaced the random systematic patch force is periodic as well since we assume that the patches are fixed in time. Therefore, the extension of Δ beyond the period of motion fails to suppress the variance of the random patch force fluctuations.

[38] Other models for the correlation function have been previously used in the literature, for example a Gaussian correlation $C_{ab}(r) = \delta_{ab} V_{rms}^2 \exp(-4r^2/\ell^2)$. This model also allows for an analytic derivation of the force power spectrum, which is different from the one corresponding to the exponential correlation. However, the constraints computed for hypothetical new forces using either model are indistinguishable for the parameters used in this work.

[39] To obtain that the power spectrum $\mathcal{F}_x[k_x]$ peaks at $k_x = 2m/R_0$ we have used $\int_{-\infty}^{\infty} dk_x \int_0^{2\pi} d\phi \mathcal{G}(k_x) \times \exp[-i(m - k_x R_0/2))\phi] \approx (4\pi/R_0)\mathcal{G}(2m/R_0)$, with $\mathcal{G}(k_x)$ given by the integrand of Eq. (4) before the k_x-integral has been performed (which is slowly varying on k_x-scales of $4\pi/R_0$) and Δ is equal to 2π.

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